ISOPERIMETRY WITHOUT CURVES OR CALCULUS.

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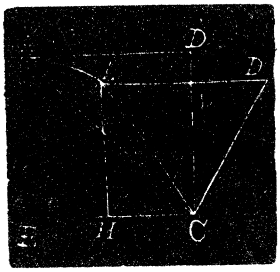
[Continued from the October Number.]

PROPOSITION III. *Of all triangles formed with two given sides, that in which these sides are perpendicular to each other is a maximum.*

Let ABC and $A'BC$ be two triangles having $AB = A'B$, BC common, and ABC a right-angle.



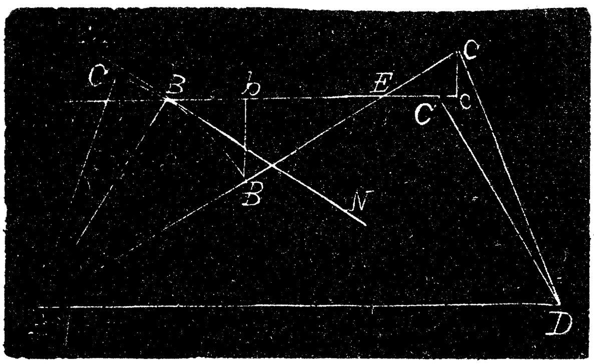
Now $A'H < A'B = AB$. Hence the bases being the same, and the altitude of the one less than the altitude of the other, the area is also less.

PROPOSITION IV. *Any rectangle is greater than any rhomboid, if their bases and perimeters are equal.*

Completing the parallelograms $ABCD$ and $A'BCD'$, we have, $ABCD > A'BCD'$, since they are respectively double the triangles ABC and $A'BC$.

COROLLARY. It follows that any square is greater than any rhombus of equal perimeter.

PROPOSITION V. *Of all quadrilaterals having the same sides, three being equal; that which has equal angles between equal sides, has the maximum area.*



Let $ABCD$ be such a quadrilateral in which $AB = BC = CD$, and $ABC = BCD$. And let us suppose those sides to take the position AB', $B'C'$ and $C'D$, the side AD remaining fixed.

Draw MBN perpendicular to AB. Drop the perpendiculars $B'b$ and $C'c$ on BC and BC prolonged, and prolong CB to the left. Make the triangle ABC'' = the triangle DCC'. Now angle $B'Bb > NBb$ and angle $C'Cc = C''BK < MBK = NBb$. ∴ $C'Cc < B'Bb$. Now $bE < B'E$ and $cE < C'E$; adding gives $bc < B'C''$ or $bC + Cc < B'C'' = BC$.

But $Bb + bc = BC$. ∴ $Bb + bC > bC + Cc$ or $Bb > Cc$. And (since $C'Cc < B'Bb$), still more is $C'c < Bb$, and $CC' < BB'$. Hence, triangle $DCC' <$ triangle ABB'. Moreover, since $cC' < bB'$; $C'E < B'E$, $cE < bE$, and still more is $CE < BE$.

Hence triangle ECC' <triangle EBB'. But, $ABCD=AB'ECD+ABB'+EBB'$ and $AB'C'D=AB'ECD+DCC'+ECC'$.

Hence $ABCD$ is greater than $AB'C'D$.

COROLLARY. It follows, that the quadrilateral of three equal sides, and maximum area, is a trapezoid; that the angles including the fourth side are also equal; that the opposite angles are supplementary; and that the trapezoid is inscriptible.

[*To be continued.*]

PROFESSOR SYLVESTER'S RECIPROCANTS.

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To those functions of the successive derivatives of y with respect to x, which preserve their form unaltered, except for dy/dx as a factor, when the independent and dependent variables x and y are interchanged, Professor Sylvester gave the name of *Reciprocants*.

According to the general theory with respect to the inversion of the indepent and dependent variable, we must have the relations:

$$\frac{dy}{dx}=1\bigg/\frac{dx}{dy};\ \frac{d^2y}{dx^2}=\frac{d}{dx}\left(1\bigg/\frac{dx}{dy}\right)=\frac{d}{dy}\left(1\bigg/\frac{dx}{dy}\right)\frac{dy}{dx}=-\frac{d^2x}{dy^2}\bigg/\left(\frac{dx}{dy}\right)^3;$$

$$\frac{d^3y}{dx^3}=\frac{d}{dy}\left[-\frac{d^2x}{dy^2}\bigg/\left(\frac{dx}{dy}\right)^3\right]\frac{dy}{dx}=-\left[\frac{dx}{dy}\cdot\frac{d^3x}{dy^3}-3\left(\frac{d^2x}{dy^2}\right)^2\right]\bigg/\left(\frac{dx}{dy}\right)^5;$$

$$\frac{d^4y}{dx^4}=-\left[\left(\frac{dx}{dy}\right)^2\left(\frac{d^4x}{dy^4}\right)-10\frac{dx}{dy}\cdot\frac{d^2x}{dy^2}\cdot\frac{d^3x}{dy^3}+15\left(\frac{d^2x}{dy^2}\right)^3\right]\bigg/\left(\frac{dx}{dy}\right)^7;\ \text{etc.}$$

After these relations are substituted for the various differential coefficients of y with respect to x, in any function of these differential coefficients or derivatives, we are said to have interchanged the independent and dependent variable.

Assume $dy/dx=T$, $d^2y/dx^2=A\,\underline{|2}$, $d^3y/dx^3=B\,\underline{|3}$, $d^4y/dx^4=C\,\underline{|4}$, etc.; then, after eliminating the constants in the general equation of the straight line, by the method of differentiation, we obtain $d^2y/dx^2=A,=d^2x/dy^2=0$ ....(1).

The left-hand member of (1) is Professor Sylvester's first *pure* reciprocant, since it does not involve dy/dx; and this reciprocant is briefly and typically expressed by A. The third member of (1) represents the reciprocant when the independent and dependent variables x and y are interchanged.